Total number of printed pages–8

3 (Sem–6) MAT M1

2020

MATHEMATICS

(Major)

Paper : 6·1

(Hydrostatics)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1\times7=7$

   (a) Which of the following is a false statement ?

   Pressure of a fluid at a given depth

   (i) is equal in all directions

   (ii) is dependent on the shape of the container

   (iii) depends on the density of the fluid

   (iv) acts in the normal direction to the surface with which it is in contact

Contd.

(b) If two volumes v_1, v_2 of different gases at pressures p_1 and p_2 and absolute temperatures T_1 and T_2 are mixed together so that the volume of the mixture is V and the absolute temperature T, then what is the pressure of the mixture?

(c) The amount of heat required to raise the temperature of a body by one degree is called

(i) thermal capacity

(ii) specific heat

(iii) isothermal temperature

(iv) None of the above

(d) Write an equation that expresses the relation between pressure and volume in adiabatic change.

(e) Define centre of pressure of a plane area immersed in a liquid at rest.

(f) If the equilibrium is unstable, what is the position of the metacentre w.r.t. the centre of gravity of the body?

(g) Which of the following is true?

Fluids —

(i) cannot be compressed

(ii) are not affected by change in temperature

(iii) are not viscous

(iv) do not offer any resistance to change in shape

2. Answer the following: 2×4=8

(a) Obtain the expression for pressure at a point in a homogeneous liquid of density ρ and acted upon by component forces per unit mass X, Y, Z.

(b) Obtain the formula for determination of the centre of pressure of any plane area in Cartesian coordinate.

(c) Define metacentre and metacentric height of a floating body.

(d) Find the work done in compressing a gas under isothermal conditions, where π is the atmospheric pressure.

3. Answer ***any three*** parts: 5×3=15

(a) A solid cone is following with its axis vertical and vertex downwards. Discuss its stability of equilibrium.

(b) Show that in a conservative field of force, the surface of equipressure, equidensity and equipotential energy coincide.

(c) A quadrant of a circle is just immersed vertically with one edge in the surface, in a liquid, the density of which varies as the depth. Find the centre of pressure.

(d) A solid body consists of a right cone joined to a hemisphere on the same base and floats with the spherical portion partly immersed. Prove that greatest height of the cone consistent with stability is $\sqrt{3}$ times of the radius of the base.

(e) For a perfect gas establish the relation $C_p - C_v = R$, where symbols have their usual meanings.

4. Answer ***any one*** part :

(a) (i) Show that the forces represented by
$$X = \mu\left(y^2 + z^2 + yz\right),$$
$$Y = \mu\left(z^2 + x^2 + zx\right),$$
$$Z = \mu\left(x^2 + y^2 + xy\right)$$
will keep a mass of liquid at rest, if the density $\propto \dfrac{1}{(\text{distance})^2}$ from the plane $x + y + z = 0$; and the curves of equal pressure and density will be circles. 5

(ii) A circular cylinder of radius a, is floating freely in water with axis vertical. At first the water was at rest and it is then made to rotate about an axis, which is the axis of the cylinder, with an angular velocity of ω. Show that in the second case an extra length $\frac{\omega^2 a^2}{4g}$ of the surface of the cylinder is wetted. 5

(b) (i) A given volume of liquid is at rest on a fixed plane under the action of a force, to a fixed point in the plane, varying as the distance. Find the pressure at any point in the liquid and the whole pressure on the fixed plane. 5

(ii) A tube in the form of a parabola held with its vertex downwards and axis vertical, is filled with two different liquids of densities δ and δ'. If the distances of the free surfaces of the liquids from the focus be r and r' respectively, show that the distance of their common surface from focus is

$$\frac{r\delta - r'\delta'}{\delta - \delta'}.$$ 5

5. Answer ***any one*** part :

(a) (i) Show that the depth of the centre of pressure of the area included between the arc and the asymptote of the curve $(r-a)\cos\theta = b$ is $\frac{a}{4}\times\frac{3\pi a+16b}{3\pi b+4a}$, the asymptote being in the surface and the plane of the curve vertical. 5

(ii) A hemispherical bowl is filled with water. Find the horizontal thrust on one-half of the surface divided by a vertical diameter plane and show that it is $\frac{1}{\pi}$ times the magnitude of the resultant fluid thrust on the whole surface. 5

(b) (i) A solid displaces $\frac{1}{a}, \frac{1}{b}$ and $\frac{1}{c}$ of its volumes, respectively when it floats in three different liquids. Find what fraction of its volume it displaces when it floats in a mixture formed of equal volumes of three liquids. 5

(ii) A quadrant of a circle is just increased vertically with one edge in the surface, in a liquid, the density of which varies as the depth. Determine the centre of pressure. 5

6. Answer ***any one*** part :

(a) (i) A solid cone of semivertical angle α, specific gravity σ floats in equilibrium in the liquid of specific gravity ρ with its axis vertical and vertex downwards. Show that the equilibrium is stable if $\frac{\sigma}{\rho} > \cos^6 \alpha$. 5

(ii) A bent tube of uniform bore, the arms of which are at right angles, revolves with constant angular velocity ω about the axis of one of its arms, which is vertical and has its extremity immersed in water. Prove that the height to which the water will rise in the vertical arm

is $\frac{\pi}{ge}\left[1 - e^{\frac{-\omega^2 a^2}{2k}}\right]$, a being the length of the horizontal arm, π the atmospheric pressure, ρ the density of water and k the ratio of the pressure of the atmosphere to its density. 5

(b) (i) The height of the Torricellian vacuum in a barometer is a inches and the instrument indicates a pressure of b inches of mercury when the true reading is c inches. If the faulty readings are due to an imperfect vacuum, prove that the true reading corresponding to an apparent reading of d inches is

$$d+\frac{a(c-d)}{a+b-d}.$$ 5

(ii) In convective equilibrium, show that the relation between absolute temperature T and height of the homogeneous atmosphere H is

$$T=T_0\left[1-\left(\frac{\gamma-1}{\gamma}\right)\frac{Z}{H}\right],$$

when the acceleration due to gravity is constant. 5
