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3 (Sem–5/CBCS) STA HC 1

2021

(Held in 2022)

STATISTICS

(Honours)

Paper : STA–HC–5016

(Stochastic Processes and Queuing Theory)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following as directed : 1×7=7

   (a) The mean of X in terms of the probability generating function (p.g.f.) of X is given by

   (i) $P''(1)$

   (ii) $P'(1)$

   (iii) $P'(S)$

   (iv) None of the above

   (Choose the correct option)

Contd.

(b) Define stochastic processes.

(c) An irreducible Markov chain contains

(i) one closed set

(ii) two closed sets

(iii) three closed sets

(iv) All of the above

(Choose the correct option)

(d) Difference of two independent Poisson processes is also a Poisson process.

*(State True **or** False)*

(e) State *one* property of transition probability matrix.

(f) The average queue size in M/M/1/1 queueing model is

(i) 1

(ii) 0

(iii) 2

(iv) None of the above

(Choose the correct option)

(g) The interval between two successive occurrences of a Poisson process has an ________ distribution.

(Fill in the blank)

2. Answer the following questions briefly : 2×4=8

(a) Define bivariate probability generating function of a pair of random variables X and Y.

(b) Define Markov chain with an example.

(c) State *any two* postulates for Poisson process.

(d) Define traffic intensity. State the condition for existence of steady state of M/M/1 queuing model with infinite system capacity. 1+1=2

3. Answer ***any three*** of the following questions: 5×3=15

(a) Let X be a Poisson variate with p.m.f

$$p_k = P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!};\ k = 0, 1, 2, \ldots.$$

Find the mean and variance of X using probability generating function (p.g.f) of X.

(b) Write an explanatory note on specification of stochastic processes.

(c) Derive the probability distribution of number of customers in M/M/1 queuing model with finite system capacity.

(d) Suppose that the probability of a dry day (state 0) following a rainy day (state 1) is $\frac{1}{3}$ and that the probability of a rainy day following a dry day is $\frac{1}{2}$. Given that May 1 is a dry day, find the probability that May 3 is also a dry day.

(e) Suppose that customers arrive at a service counter in accordance with a Poisson process with mean rate of 2 per minute ($\lambda = 2$/minute). Then the interval between any two successive arrivals follow exponential distribution with mean $1/\lambda = 1/2$ minute. Find the probability that the interval between two successive arrivals is —

(i) more than 1 minute;

(ii) 4 minutes or less;

(iii) between 1 and 2 minutes.

4. Answer ***either*** *(a)* **or** *(b)* :

(a) *(i)* Define matrix of transition probabilities. Let $\{X_t, t \geq 0\}$ be a Markov chain with three states 0, 1 and 2 with transition matrix

$$\begin{pmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{pmatrix}$$

and the initial distribution

$$P\{X_0 = i\} = \frac{1}{3}, \quad i = 0, 1, 2$$

Find $P\{X_1 = 1/X_0 = 2\}$;
$P\{X_2 = 2, X_1 = 1/X_0 = 2\}$ and
$P\{X_2 = 2, X_1 = 1, X_0 = 2\}$.

1+1+2+2=6

(ii) Write a short note on Markov chain as graphs. 4

(b) (i) Define an irreducible Markov chain. Prove (or disprove) that the matrix given below is a transition probability matrix of an irreducible Markov chain : 1+5=6

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$$

(ii) Prove the additive property of Poisson process. 4

5. Answer ***either*** (a) ***or*** (b) :

(a) The arrivals at a counter in a bank occur in accordance with a Poisson process at an average arrival rate of 8/hour. The duration of service of a customer follows exponential distribution with a mean rate of 6 minutes. Find the following :

(i) The probability that an arriving customer has to wait.

(ii) The average number of customers in the queue.

(iii) The average number of customers in the system.

(iv) The probability that there are two customers in the system.

(v) The average waiting time in the queue.

2+2+2+2+2=10

(b) (i) Define the following states of Markov chain : 1+1+1+1=4
Absorbing state, Persistant state, Transient state and Ergodic state

(ii) Given the following transition probability matrix

$$P = \begin{pmatrix} q & p \\ p & q \end{pmatrix} \text{ where } p + q = 1$$

Find the probability of transition from state 0 to state 1 in m steps. 6

6. Answer ***either*** *(a)* ***or*** *(b)* :

(a) (i) Differentiate between steady state and transient state of a queuing system. 2

(ii) Define a stationary process. 2

(iii) Write an explanatory note on basic characteristics of a queuing system. 6

(b) (i) Write a detailed note on applications of stochastic processes. 5

(ii) If $\{N(t)\}$ is a Poisson process and $s < t$, then prove that

$$P\{N(s)=k/N(t)=n\}=\binom{n}{k}\left(\frac{s}{t}\right)^k\left(1-\frac{s}{t}\right)^{n-k}$$

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