Total number of printed pages-4

PG/Sem-1(Ar/Bt)/MAT-103

2022

Mathematics

Paper : MAT-1036

(Mechanics and Tensor)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions.

(Use separate khatas for each half)

FIRST HALF

(Mechanics)

Marks : 50

1. Answer *any one* of the following : 10

a) A particle moves on the inner surface of a smooth cone, of vertical angle 2α , being acted on by a force towards the vertex of the cone, and its direction of motion always cuts the generators at a constant angle β. Find the motion and the law of force.

b) A particle moves on a smooth sphere under no forces except the pressure of the surface; show that its path is given by the equation $cot\theta = cot\beta \cos\phi$, where θ and ϕ are its angular constants.

2. Answer *any one* of the following : 8

a) A rectangular lamina, whose sides are of length *2a* and *2b*, is at

Contd.

rest when one corner is caught and suddenly made to move with prescribed speed V in the plane of the lamina. Show by using Kelvin's theorem that the greater angular velocity which can thus be imparted to the lamina is $\frac{3V}{4\sqrt{a^2+b^2}}$.

b) Two equal uniform rods, AB and AC, are freely jointed at A and are placed on a smooth table so as to be at right angles. The rod AC is stuck by a blow at C in a direction perpendicular to itself. Show that the resulting velocities of the middle points of AB and AC are in the ratio 2 : 7.

3. Answer *any two* parts : 6×2=12

a) Establish Euler's dynamical equations of motion of a rigid body with one point fixed,where the axes are not necessarily the principal axes.

b) If T is the total kinetic energy of rotation of a rigid body with one point fixed, prove that $\frac{dT}{dt} = \vec{\omega}.\vec{\Lambda}$ where all quantities refer to the principal axes of the body.

c) A rigid body having one point O fixed and no external torque about O has two equal principal moments of inertia. Prove that it must rotate with angular velocity of constant magnitude.

4. Answer *any two* parts : 5×2=10

a) With usual notations, derive Lagrange's equations for conservative field of forces in the form

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_\alpha}\right)=\frac{\partial L}{\partial q_\alpha}, \ \alpha = 1,2,3\ldots\ldots, \text{ where } L = T - V.$$

b) Use Lagrange's equation to discuss the motion of a particle in a plane under an attractive central force obeying the law of inverse square.

c) A bead slides without friction on a frictionless wire in the shape of a cycloid with equations

$x = a(\theta - \sin\theta),\ y = a(1+\cos\theta);\ 0 \le \theta \le 2\pi$

Obtain the Lagrangian for the motion of the bead.

5. Answer *any one* : 10

a) Define the Hamiltonian and write the Hamiltonian function for the motion of a compound pendulum and hence obtain the equation of motion for a compound pendulum.

b) If $H(p,q) = \frac{1}{2}(p^2 + \lambda^2 q^2)$ and q and p are transformed into Q, P by means of relations

$$q = (2Q)^{\frac{1}{2}} \lambda^{-\frac{1}{2}} \cos P,$$

$$p = (2Q)^{\frac{1}{2}} \sin P;$$

show that the system of Hamilton's equation in (q, p) is changed again into another such system with the Hamiltonian equal to λQ.

SECOND HALF

(Tensor)

Marks : 30

1. Answer ***any three*** parts of the following: 5×3=15

a) Define symmetric and anti-symmetric tensors. Show that every second order tensor (covariant or contravariant) can be expressed as a sum of symmetric and anti-symmetric tensors.

b) State and prove the quotient theorem of tensors.

c) Establish the transformation law of the Christoffel symbol of the first kind.

d) Show that the functions $g_{\mu\nu}$ in the metric $ds^2 = g_{\mu\nu}\, dx^{\mu} dx^{\nu}$ is a symmetric covariant tensor of rank two.

2. Answer ***any three*** parts of the following : 5×3=15

a) Derive the x^j covariant derivative of the covariant tensor A_i of rank one with respect to the fundamental tensor g_{ij}.

b) Show that the covariant derivatives of the tensors g_{ij}, g^{ij}, δ^i_j, all vanish identically.

c) Show that the angle between the coordinate curves x^i = constant and x^j = constant is given by $\cos\theta = \dfrac{g^{ij}}{\sqrt{g^{ii}}\sqrt{g^{jj}}}$

d) Show that the covariant derivative of a scalar invariant function ϕ is equal to its partial derivative.

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