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3 (Sem-4/CBCS) STA HC 1

2023

STATISTICS

(Honours Core)

Paper : STA-HC-4016

(Statistical Inference)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following as directed : $1 \times 7 = 7$

(a) Sample median is _____ estimator for the mean of normal population.

(Fill in the blank)

(b) Unbiased estimators are necessarily consistent.

(State True or False)

Contd.

(c) Area of critical region depends on

(i) number of observations

(ii) value of the statistic

(iii) size of type I error

(iv) size of type II error

(Choose the correct option)

(d) For a certain test if $\alpha = 0.05$,
 $\beta = 0.10$, then the power of the test is

(i) 0.95

(ii) 0.90

(iii) 0.05

(iv) 0.10

(Choose the correct option)

(e) Sample moments are _____ estimators
of the corresponding population
moments. *(Fill in the blank)*

(f) Suppose we put forward an interval which we expect to include the true parameter value, then the process is called _____ estimation.

(Fill in the blank)

(g) The Neyman-Pearson lemma proceeds to find the best critical region for testing _____ hypothesis against _____ alternative hypothesis.

(Fill in the blanks)

2. Answer the following questions : $2 \times 4 = 8$

(a) If x_1, x_2, \dots, x_n is a random sample from a normal population $N(\mu, 1)$, then

show that $T = \sum_{i=1}^n x_i^2$ is an unbiased

estimator of $\mu^2 + 1$.

(b) Find the maximum likelihood estimator of θ for the following probability distribution :

$$f(x, \theta) = \theta e^{-\theta x}, \quad x > 0, \theta > 0$$

- (c) State the Neyman-Pearson lemma.
- (d) Give example of a maximum likelihood estimator which is not unbiased.

3. Answer **any three** questions from the following : 5×3=15

- (a) Obtain the M.L.E. of α and β for the rectangular distribution

$$f(x : \alpha, \beta) = \begin{cases} \frac{1}{\beta - \alpha}, & \alpha < x < \beta \\ 0, & \text{elsewhere} \end{cases}$$

- (b) Show that, if a sufficient estimator exists, it is a function of the M.L.E.
- (c) What is meant by statistical hypothesis? Explain the concept of type I and type II error with example. What is the power of a test ?

(d) Let X have the p.d.f. of the form

$$f(x, \theta) = \theta x^{\theta-1}, 0 < x < 1 \\ = 0, \text{ elsewhere}$$

Find the most powerful test to test the simple hypothesis

$$H_0 : \theta = 1$$

against the alternative hypothesis

$$H_1 : \theta = 2$$

by means of a single observation X . What would be the size of type I and type II error, if you choose the interval

(i) $x \geq 0.05$

(ii) $x \geq 1.5$

as critical region ?

(e) Let x_1, x_2, \dots, x_n be a random sample from a distribution with p.d.f.

$$f(x, \theta) = e^{-(x-\theta)}, \theta < x < \infty \\ -\infty < \theta < \infty$$

Obtain a sufficient statistic for θ .

4. Answer **any three** questions from the following : 10×3=30

(a) What do you mean by MP and UMP tests ? Show that the most powerful test is necessarily unbiased.

(b) State the Cramer-Rao inequality. What are the conditions for equality sign in C-R inequality ? Show that,

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

in random sampling from

$$f(x, \theta) = \begin{cases} \frac{1}{\theta} \exp(-x/\theta), & 0 < x < \infty \\ 0, & \text{elsewhere} \end{cases}$$

where, $0 < \theta < \infty$ is an MVB estimator

of θ and has variance $\frac{\theta^2}{n}$.

(c) Define consistent estimator. State and prove the sufficient condition for consistency of an estimator.

(d) Show that with the help of example,

(i) an MLE is not unique;

(ii) an MLE may not exist.

- (e) What is likelihood ratio test ? Show that likelihood ratio test for testing the variances of two normal population is the usual F-test.
- (f) (i) Describe the method of moments for estimating parameter.
- (ii) Show that in sampling from Cauchy population,

$$f(x, \theta) = \frac{1}{\pi [1 + (x - \theta)^2]}, \quad \begin{matrix} -\infty < x < \infty \\ \theta > 0 \end{matrix}$$

is not sample mean, but sample median is a consistent estimator of θ .
